Available online at www.ijpab.com

DOI: http://dx.doi.org/10.18782/2582-2845.8203

ISSN: 2582 – 2845

Ind. J. Pure App. Biosci. (2020) *8*(4), 257-266

Indian Journal of Pure & Applied Biosciences

Peer-Reviewed, Refereed, Open Access Journal

Research Article

Alternative Ratio Estimators for Estimating Population Mean in Simple Random Sampling Using Auxiliary Information

S. Maqbool[1*], Mir Subzar[1] and Shakeel Javaid[2]

[1]Division of AGB, FVSC&AH, SHUHAMA, SKUAST-K

[2]Deptt. of Statistics & OR, AMU, Aligarh

*Corresponding Author E-mail: showkatmaq@gmail.com

Received: 4.06.2020 | Revised: 9.07.2020 | Accepted: 14.07.2020

ABSTRACT

Alternative ratio estimators are proposed for a finite population mean of a study variable in simple random sampling using the information on the mean of the auxiliary variable, which is positively correlated with the study variable. The properties associated with the proposed estimators are assessed by mean square error and bias and the expressions for bias and mean square for proposed estimators are also obtained. Both analytical and numerical comparisons have shown the proposed alternative estimators are more efficient than the classical ratio and the existing estimators under consideration.

Keywords: *Ratio estimators, Auxiliary information, Bias, Mean Square Error, Simple Random Sampling, Efficiency.*

AMS Subject Classification: *62D05*

INTRODUCTION

In Sample Surveys, auxiliary information is always used to improve the precision of the estimates of the population parameters. This can be done at either estimation or selection stage or both stages. The commonly used estimators, which make use of auxiliary variables, include ratio estimators, regression estimator, product estimator and difference estimator. The classical ratio estimator is preferred when there is a high positive correlation between the variable of interest, Y and the auxiliary variable, X with the regression line passing through the origin. The classical product estimator, on the other hand is mostly preferred when there is a high negative correlation between Y and X while the linear regression estimator is most preferred when there is high positive correlation between the two variables and the regression line of the study variable on the auxiliary variable has intercept on Y axis. Ratio estimation has gained relevance in estimation theory because of its improved precision in estimating the population parameters.

Cite this article: Maqbool, S., Subzar, M., & Javaid, S. (2020). Alternative Ratio Estimators for Estimating Population Mean in Simple Random Sampling Using Auxiliary Information, *Ind. J. Pure App. Biosci.* *8*(4), 257-266. doi: http://dx.doi.org/10.18782/2582-2845.8203

It has been widely applied in Agriculture to estimate the mean yield of crops in a certain area and in Forestry, to estimate with high precision, the mean number of trees or crops in a forest or plantation. Other areas of relevance include Economics and Population studies to estimate the ratio of the income to family size.

So Cochran (1940) initiated the use of auxiliary information at estimation stage and proposed ratio estimator for population mean. It is well established fact that ratio type estimators provide better efficiency in comparison to simple mean estimator if the study variable and auxiliary variable are positively correlated and the regression line pass through origin and if on the other side correlation between the study variable and auxiliary variable is positive and does not pass through origin, but makes an intercept, in that case regression method provide better efficiency than ratio, simple mean and product type estimator and if the correlation between the study variable and auxiliary variable is negative, product estimator given by Robson (1957) is more efficient than simple mean estimator.

Further improvements are also achieved on the classical ratio estimator by introducing a large number of modified ratio estimators with the use of known parameters like, coefficient of variation, coefficient of kurtosis, coefficient of skewness and population correlation coefficient. For more detailed discussion one may refer to Cochran (1977), Kadilar and Cingi (2004, 2006), Koyuncu and Kadilar (2009), Murthy (1967), Prasad (1989), Rao (1991), Singh (2003), Singh and Tailor (2003, 2005), Singh et al. (2004), Sisodia and Dwivedi (1981), Upadhyaya and Singh (1999) and Yan and Tian (2010).

Further, Subramani and Kumarapandiyan (2012) had taken initiative by proposing modified ratio estimator for estimating the population mean of the study variable by using the population deciles of the auxiliary variable.

Recently Subzar et al. (2016) had proposed some estimators using population deciles and correlation coefficient of the auxiliary variable, also Subzar et al. (2017) had proposed some modified ratio type estimators using the quartile deviation and population deciles of auxiliary variable and Subzar et al. (2017) had also proposed an efficient class of estimators by using the auxiliary information of population deciles, median and their linear combination with correlation coefficient and coefficient of variation and Subzar et al. (2017) also proposed some modified ratio estimators for estimating population mean using the auxiliary information of quartiles and their linear combination with correlation coefficient and coefficient of variation.

In this paper we have envisaged an alternative ratio estimator's for estimation of population mean of the study variable using the information of non-conventional location parameters, non-conventional measures of dispersion, coefficient of variation and median. Let $G = \{G_1, G_2, G_3, ..., G_N\}$ be a finite population of N distinct and identifiable units. Let y and x denotes the study variable and the auxiliary variable taking values y_i and x_i respectively on the i^{th} unit ($i = 1, 2,…,N$).

The classical Ratio estimator for the population mean $\bar{Y}$ of the study variable Y is defined as:

$$\hat{\bar{Y}}_R = \frac{\bar{y}}{\bar{x}} \bar{X} = \hat{R}\bar{X}, \text{ where } \hat{R} = \frac{\bar{y}}{\bar{x}}$$

Where $\bar{y}$ is the sample mean of the study variable Y and $\bar{x}$ is the sample mean of the auxiliary variable X . It is assumed that the population mean $\bar{X}$ of the auxiliary variable X is known. The bias and mean squared error of $\hat{\bar{Y}}_R$ to thefirst degree of approximation are given below;

$$B(\hat{\bar{Y}}_R) = \frac{(1-f)}{n} \bar{Y} (C_x^2 - C_x C_y \rho)$$

$$MSE(\hat{\bar{Y}}_R) = \frac{(1-f)}{n} \bar{Y}^2 (C_y^2 + C_x^2 - 2C_x C_y \rho)$$

Before discussing about the proposed estimators, we will mention the estimators in Literature using the notations given in the next sub-section.

1.1. Notations

N	Population size
n	Sample size
$f = n/N$	Sampling fraction
Y	Study variable
X	Auxiliary variable
$\bar{X}, \bar{Y}$	Population means
$\bar{x}, \bar{y}$	Sample means
x, y	Sample totals
S_x, S_y	Population standard deviations
S_{xy}	Population covariance between variables
C_x, C_y	Population coefficient of variation
ρ	Population correlation coefficient
$B(.)$	Bias of the estimator
$MSE(.)$	Mean square error of the estimator
$\hat{\bar{Y}}_i$	Existing modified ratio estimator of $\bar{Y}$
$\hat{\bar{Y}}_{pj}$	Proposed modified ratio estimator of $\bar{Y}$
M_d	Population median of X
β_2	Population kurtosis
β_1	Population skewness

$TM = \frac{Q_1 + 2Q_2 + Q_3}{4}$ Tri-Mean

$HL = median((X_j + X_k)/2, 1 \le j \le k \le N)$ Hodges-Lehmann estimator

$MR = \frac{X_{(1)} + X_{(N)}}{2}$ Population mid-range

$G = \frac{4}{N-1}\sum_{i=1}^{N}\left(\frac{2i-N-1}{2N}\right)X_{(i)}$ Gini's Mean Difference

$D = \frac{2\sqrt{\pi}}{N(N-1)}\sum_{i=1}^{N}\left(i - \frac{N+1}{2}\right)X_{(i)}$ Downton's method

$S_{pw} = \frac{\sqrt{\pi}}{N^2}\sum_{i=1}^{N}(2i-N-1)X_{(i)}$ Probability weighted moments

$DM = \frac{D_1 + D_2 + ... + D_9}{9}$ Decile mean

Subscript

i For existing estimators j For proposed estimators

1.2. Estimators in Literature

Abid et al. (2016) suggested the following ratio estimators for the population mean $\bar{Y}$ in simple random sampling using non-conventional location parameters as auxiliary information. Estimators suggested by Abid et al. (2016) are given as:

$$\hat{\bar{Y}}_1 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+TM)}(\bar{X}+TM), \qquad \hat{\bar{Y}}_2 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+TM)}(\bar{X}C_x+TM)$$

$$\hat{\bar{Y}}_3 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+TM)}(\bar{X}\rho+TM), \qquad \hat{\bar{Y}}_4 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+MR)}(\bar{X}+MR),$$

$$\hat{\bar{Y}}_5 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+MR)}(\bar{X}C_x+MR), \qquad \hat{\bar{Y}}_6 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+MR)}(\bar{X}\rho+MR)$$

$$\hat{\bar{Y}}_7 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+HL)}(\bar{X}+HL), \qquad \hat{\bar{Y}}_8 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+HL)}(\bar{X}C_x+HL),$$

$$\hat{\bar{Y}}_9 = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+HL)}(\bar{X}\rho+HL)$$

The biases, related constants and the mean square error (MSE) for Abid et al. (2016) estimators are respectively given by:

$$B(\hat{\bar{Y}}_1) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_1^2, \qquad R_1 = \frac{\bar{Y}}{(\bar{X}+TM)} \qquad MSE(\hat{\bar{Y}}_1) = \frac{(1-f)}{n}(R_1^2S_x^2+S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_2) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_2^2, \qquad R_2 = \frac{\bar{Y}C_x}{(\bar{X}C_x+TM)} \qquad MSE(\hat{\bar{Y}}_2) = \frac{(1-f)}{n}(R_2^2S_x^2+S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_3) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_3^2, \qquad R_3 = \frac{\bar{Y}\rho}{(\bar{X}\rho+TM)} \qquad MSE(\hat{\bar{Y}}_3) = \frac{(1-f)}{n}(R_3^2S_x^2+S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_4) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_4^2, \qquad R_4 = \frac{\bar{Y}}{(\bar{X}+MR)} \qquad MSE(\hat{\bar{Y}}_4) = \frac{(1-f)}{n}(R_4^2S_x^2+S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_5) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_5^2, \qquad R_5 = \frac{\bar{Y}C_x}{(\bar{X}C_x+MR)} \qquad MSE(\hat{\bar{Y}}_5) = \frac{(1-f)}{n}(R_5^2S_x^2+S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_6) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_6^2, \qquad R_6 = \frac{\bar{Y}\rho}{(\bar{X}\rho+MR)} \qquad MSE(\hat{\bar{Y}}_6) = \frac{(1-f)}{n}(R_6^2S_x^2+S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_7) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_7^2, \qquad R_7 = \frac{\bar{Y}}{(\bar{X}+HL)} \qquad MSE(\hat{\bar{Y}}_7) = \frac{(1-f)}{n}(R_7^2S_x^2+S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_8) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_8^2, \qquad R_8 = \frac{\bar{Y}C_x}{(\bar{X}C_x+HL)} \qquad MSE(\hat{\bar{Y}}_8) = \frac{(1-f)}{n}(R_8^2S_x^2+S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_9) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_9^2, \qquad R_9 = \frac{\bar{Y}\rho}{(\bar{X}\rho+HL)} \qquad MSE(\hat{\bar{Y}}_9) = \frac{(1-f)}{n}(R_9^2S_x^2+S_y^2(1-\rho^2)).$$

Abid et al. (2016) suggested the following ratio estimators for the population mean $\bar{Y}$ in simple random sampling using Decile mean, with linear combination of population correlation coefficient and population coefficient of variation as auxiliary information. Estimators suggested by Abid et al. (2016) are given as:

$$\hat{\hat{Y}}_{10}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+DM)}(\bar{X}+DM)\ \hat{\hat{Y}}_{11}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+DM)}(\bar{X}C_x+DM)$$

$$\hat{\hat{Y}}_{12}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+DM)}(\bar{X}\rho+DM)$$

The biases, related constants and the mean square error (MSE) for Abid *et al* (2016) estimators are respectively given by:

$$B(\hat{\hat{Y}}_{10})=\frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{10}^2,\qquad R_{10}=\frac{\bar{Y}}{(\bar{X}+DM)}\qquad MSE(\hat{\hat{Y}}_{10})=\frac{(1-f)}{n}(R_{10}^2S_x^2+S_y^2(1-\rho^2)),$$

$$B(\hat{\hat{Y}}_{11})=\frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{11}^2,\qquad R_{11}=\frac{\bar{Y}C_x}{(\bar{X}C_x+DM)}\qquad MSE(\hat{\hat{Y}}_{11})=\frac{(1-f)}{n}(R_{11}^2S_x^2+S_y^2(1-\rho^2)),$$

$$B(\hat{\hat{Y}}_{12})=\frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{12}^2,\qquad R_{12}=\frac{\bar{Y}\rho}{(\bar{X}\rho+DM)}\qquad MSE(\hat{\hat{Y}}_{12})=\frac{(1-f)}{n}(R_{12}^2S_x^2+S_y^2(1-\rho^2)).$$

1. IMPROVED RATIO ESTIMATORS

Motivated by the mentioned estimators in Section 1.2, we propose Alternative ratio estimators using the linear combination of non-conventional location parameters, non-conventional measures of dispersion with coefficient of variation and median.

$$\hat{\hat{Y}}_{p1}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\gamma\bar{x}+TM)}(\gamma\bar{X}+TM),\ \hat{\hat{Y}}_{p2}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\gamma\bar{x}+MR)}(\gamma\bar{X}+MR),$$

$$\hat{\hat{Y}}_{p3}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\gamma\bar{x}+HL)}(\gamma\bar{X}+HL),\ \hat{\hat{Y}}_{p4}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\gamma\bar{x}+G)}(\gamma\bar{X}+G),\ \hat{\hat{Y}}_{p5}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\gamma\bar{x}+D)}(\gamma\bar{X}+D),$$

$$\hat{\hat{Y}}_{p6}=\frac{\bar{y}+b(\bar{X}-\bar{x})}{(\gamma\bar{x}+S_{pw})}(\gamma\bar{X}+S_{pw}),$$

Where $\gamma=C_x/M_d$.

The bias, related constant and the MSE for the first proposed estimator can be obtained as follows:

MSE of this estimator can be found using Taylor series method defined as

$$h(\bar{x},\bar{y})\cong h(\bar{X},\bar{Y})+\frac{\partial h(c,d)}{\partial c}|_{\bar{X},\bar{Y}}\ (\bar{x}-\bar{X})+\frac{\partial h(c,d)}{\partial d}|_{\bar{X},\bar{Y}}\ (\bar{y}-\bar{Y})$$

(2.1)

Where $h(\bar{x},\bar{y})=\hat{R}_{p1}$ and $h(\bar{X},\bar{Y})=R.$

As shown in Wolter (1981), (2.1) can be applied to the proposed estimator in order to obtain MSE equation as follows:

$$\hat{R}_{p1}-R\cong\frac{\partial((\bar{y}+b(\bar{X}-\bar{x}))/(\gamma\bar{x}+TM)}{\partial\bar{x}}|_{\bar{X},\bar{Y}}\ (\bar{x}-\bar{X})+\frac{\partial((\bar{y}+b(\bar{X}-\bar{x}))/(\gamma\bar{x}+TM)}{\partial\bar{y}}|_{\bar{X},\bar{Y}}\ (\bar{y}-\bar{Y})$$

$$\cong-\left(\frac{\bar{y}}{(\gamma\bar{x}+TM)^2}+\frac{b(\gamma\bar{X}+TM)}{(\gamma\bar{x}+TM)^2}\right)|_{\bar{X},\bar{Y}}\ (\bar{x}-\bar{X})+\frac{1}{(\gamma\bar{x}+TM)}|_{\bar{X},\bar{Y}}\ (\bar{y}-\bar{Y})$$

$$E(\hat{R}_{p1}-R)^2\cong\frac{(\gamma\bar{Y}+B(\gamma\bar{X}+TM))^2}{(\gamma\bar{X}+TM)^4}V(\bar{x})-\frac{2(\gamma\bar{Y}+B(\gamma\bar{X}+TM))}{(\gamma\bar{X}+TM)^3}Cov(\bar{x},\bar{y})+\frac{1}{(\gamma\bar{X}+TM)^2}V(\bar{y})$$

$$\cong\frac{1}{(\gamma\bar{X}+TM)^2}\left\{\frac{(\gamma\bar{Y}+B(\gamma\bar{X}+TM))^2}{(\gamma\bar{X}+TM)^2}V(\bar{x})-\frac{2(\gamma\bar{Y}+B(\gamma\bar{X}+TM)}{(\gamma\bar{X}+TM)}Cov(\bar{x},\bar{y})+V(\bar{y})\right\}$$

Where $B = \frac{s_{xy}}{s_x^2} = \frac{\rho s_x s_y}{s_x^2} = \frac{\rho s_y}{s_x}$. Note that we omit the difference of $(E(b) - B)$ v

$$MSE(\bar{y}_{p1}) = (\gamma\bar{X} + TM)^2 E(\hat{R}_{p1} - R)^2 \cong \frac{(\gamma\bar{Y} + B(\gamma\bar{X} + TM))^2}{(\gamma\bar{X} + TM)^2} V(\bar{x}) - \frac{2(\gamma\bar{Y} + B(\gamma\bar{X} + TM))}{(\gamma\bar{X} + TM)} Cov(\bar{x}, \bar{y}) + V(\bar{y})$$

$$\cong \frac{\gamma\bar{Y}^2 + 2B(\gamma\bar{X} + TM)\bar{Y} + B^2(\gamma\bar{X} + TM)^2}{(\gamma\bar{X} + TM)^2} V(\bar{x}) - \frac{2\gamma\bar{Y} + 2B(\gamma\bar{X} + TM)}{(\gamma\bar{X} + TM)} Cov(\bar{x}, \bar{y}) + V(\bar{y})$$

$$\cong \frac{(1-f)}{n}\left\{\left(\frac{\gamma\bar{Y}^2}{(\gamma\bar{X} + TM)^2} + \frac{2B\gamma\bar{Y}}{(\gamma\bar{X} + TM)} + B^2\right)S_x^2 - \left(\frac{2\gamma\bar{Y}}{(\gamma\bar{X} + TM)} + 2B\right)S_{xy} + S_y^2\right\}$$

$$\cong \frac{(1-f)}{n}(R^2 S_x^2 + 2BRS_x^2 + B^2 S_x^2 - 2RS_{xy} - 2BS_{xy} + S_y^2)$$

$$MSE(\bar{y}_{p1}) \cong \frac{(1-f)}{n}(R^2 S_x^2 + 2R\rho S_x S_y + \rho^2 S_y^2 - 2R\rho S_x S_y - 2\rho^2 S_y^2 + S_y^2$$

$$\cong \frac{(1-f)}{n}(R^2 S_x^2 - \rho^2 S_y^2 + S_y^2) \cong \frac{(1-f)}{n}(R^2 S_x^2 + S_y^2(1-\rho^2)$$

Similarly, the bias is obtained as

$$Bias(\bar{y}_{p1}) \cong \frac{(1-f)}{n}\frac{S_x^2}{\bar{Y}} R_1^2$$

Thus the bias and MSE of the proposed estimator is given below:

$$B(\hat{\bar{Y}}_{p1}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}} R_1^2, \quad R_1 = \frac{\bar{Y}\gamma}{\bar{X}\gamma + TM} \qquad MSE(\hat{\bar{Y}}_{p1}) = \frac{(1-f)}{n}(R_1^2 S_x^2 + S_y^2(1-\rho^2)),$$

Similarly, the bias, constant and the mean square error can be found using the Taylor series method and is given as below:

$$B(\hat{\bar{Y}}_{p2}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}} R_2^2, \quad R_2 = \frac{\bar{Y}\gamma}{\bar{X}\gamma + MR} \qquad MSE(\hat{\bar{Y}}_{p2}) = \frac{(1-f)}{n}(R_2^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_{p3}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}} R_3^2, \quad R_3 = \frac{\bar{Y}\gamma}{\bar{X}\gamma + HL} \qquad MSE(\hat{\bar{Y}}_{p3}) = \frac{(1-f)}{n}(R_3^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_{p4}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}} R_4^2, \quad R_4 = \frac{\bar{Y}\gamma}{\bar{X}\gamma + G} \qquad MSE(\hat{\bar{Y}}_{p4}) = \frac{(1-f)}{n}(R_4^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_{p5}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}} R_5^2, \quad R_5 = \frac{\bar{Y}\gamma}{\bar{X}\gamma + D} \qquad MSE(\hat{\bar{Y}}_{p5}) = \frac{(1-f)}{n}(R_5^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{\bar{Y}}_{p6}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}} R_6^2, \quad R_6 = \frac{\bar{Y}\gamma}{\bar{X}\gamma + S_{pw}} \qquad MSE(\hat{\bar{Y}}_{p6}) = \frac{(1-f)}{n}(R_6^2 S_x^2 + S_y^2(1-\rho^2)).$$

3. Efficiency Comparisons:

From the expressions of the MSE of the proposed estimators and the existing estimators, we have derived the conditions for which the proposed estimators are more efficient than the usual and existing modified ratio estimators are given as follows:

Comparison with the classical ratio estimator

Modified proposed ratio estimators are more efficient than that of the classical ratio estimator if $MSE(\hat{\bar{Y}}_{pj}) \leq MSE(\hat{\bar{Y}}_r)$,

$$\frac{(1-f)}{n}(R_{pj}^2 S_x^2 + S_y^2(1-\rho^2) \leq \frac{(1-f)}{n}(S_y^2 + R^2 S_x^2 - 2R\rho S_x S_y),$$

$$R_{pj}^2 S_x^2 - \rho^2 S_y^2 - R^2 S_x^2 + 2R\rho S_x S_y \le 0,$$

$$(\rho S_y - RS_x)^2 - R_{pj}^2 S_x^2 \ge 0,$$

$$(\rho S_y - RS_x + R_{pj}^2)(\rho S_y - RS_x - R_{pj} S_x) \ge 0.$$

Condition I: $(\rho S_y - RS_x + R_{pj} S_x) \le 0$ and $(\rho S_y - RS_x - R_{pj} S_x) \le 0$

After solving the condition I, we get

$$\left(\frac{RS_y - RS_x}{S_x}\right) \le R_{pj} \le \left(\frac{RS_x - \rho S_y}{S_x}\right).$$

Hence,

$$MSE(\hat{\bar{Y}}_{pj}) \le MSE(\hat{\bar{Y}}_r),$$

$$\left(\frac{\rho S_y - RS_x}{S_x}\right) \le R_{pj} \le \left(\frac{RS_x - \rho S_y}{S_x}\right), \quad \text{or}$$

$$\left(\frac{RS_x - \rho S_y}{S_x}\right) \le R_{pj} \le \left(\frac{\rho S_y - RS_x}{S_x}\right). \qquad \text{Where } j = 1,2,....,6.$$

Comparisons with existing ratio estimators

$$MSE(\hat{\bar{Y}}_{pj}) \le MSE(\hat{\bar{Y}}_i),$$

$$\frac{(1-f)}{n}(R_{pj}^2 S_x^2 + S_y^2(1-\rho^2)) \le \frac{(1-f)}{n}(R_i^{\,2} S_x^2 + S_y^2(1-\rho^2)),$$

$$R_{pj}^2 S_x^2 \le R_i^2 S_x^2,$$

$$R_{pj} \le R_i,$$

Where $j = 1,2,....,6.$ and $i = 1,2,....,12.$

4. APPLICATIONS

The performances of the proposed ratio estimators are evaluated and compared with the mentioned ratio estimators in Section 1.2 by using the data of the natural population. For the population we use the data of Singh and Chaudhary 1986, page 177. We apply the proposed, classical ratio and existing estimators to this data set and the data statistics of this population is given in Table 1.

From Table 2, we observe that the proposed estimators are more efficient than all of the estimators in literature as their Bias, Constant and Mean Square error are much lower than the existing estimators.

The percentage relative efficiency (PRE) of the proposed estimators (*p*), with respective to the existing estimators (*e*), is computed by

$$PRE = \frac{MSE\,of\;\; Existing\;\; Estimator}{MSE\,of\;\; propoesd\, estimator} \times 100$$

These PRE values are given in Table 3 for the population. From this table, it is clearly evident that the proposed estimators are quiet efficient with respect to the estimators in literature.

Table 1: Characteristics of these populations

Parameters	Population	Parameters	Population
N	34	β_2	0.0978
n	20	β_1	0.9782
$\bar{Y}$	856.4117	M_d	150
$\bar{X}$	208.8823	TM	162.25
ρ	0.4491	MR	284.5
S_y	733.1407	HL	190
C_y	0.8561	G	155.446
S_x	150.5059	D	140.891
C_x	0.7205	S_{pw}	199.961

Table 2: The Statistical Analysis of the Estimators for this Population

Estimators	Population		
	Constant	Bias	MSE
$\hat{\bar{Y}}_r$	4.1000	4.2701	10539.27
$\hat{\bar{Y}}_1$	2.3076	2.9001	11317.28
$\hat{\bar{Y}}_2$	1.9730	2.1201	10649.40
$\hat{\bar{Y}}_3$	1.5021	1.2290	9886.21
$\hat{\bar{Y}}_4$	1.7358	1.6410	10239.11
$\hat{\bar{Y}}_5$	1.4185	1.0960	9772.39
$\hat{\bar{Y}}_6$	1.0167	0.5630	9316.02
$\hat{\bar{Y}}_7$	2.1470	2.5101	10983.77
$\hat{\bar{Y}}_8$	1.8120	1.7880	10365.55
$\hat{\bar{Y}}_9$	1.3550	1.9801	9690.50
$\hat{\bar{Y}}_{10}$	1.9301	2.2087	10571.58
$\hat{\bar{Y}}_{11}$	1.6013	1.3964	10030.11
$\hat{\bar{Y}}_{12}$	1.1703	0.7459	9472.95
$\hat{\bar{Y}}_{p1}$	0.0252	0.00034	8834.45
$\hat{\bar{Y}}_{p2}$	0.0144	0.00011	8834.25
$\hat{\bar{Y}}_{p3}$	0.0215	0.00025	8834.36
$\hat{\bar{Y}}_{p4}$	0.0263	0.00038	8834.48
$\hat{\bar{Y}}_{p5}$	0.0290	0.00046	8834.54
$\hat{\bar{Y}}_{p6}$	0.0205	0.00023	8834.35

Table 3: PRE of the Proposed Estimators with the Estimators in Literature for population

	$\hat{\bar{Y}}_{p1}$	$\hat{\bar{Y}}_{p2}$	$\hat{\bar{Y}}_{p3}$	$\hat{\bar{Y}}_{p4}$	$\hat{\bar{Y}}_{p5}$	$\hat{\bar{Y}}_{p6}$
$\hat{\bar{Y}}_r$	119.2974	119.3001	119.2986	119.2970	119.2962	119.2988
$\hat{\bar{Y}}_1$	128.1040	128.1069	128.1053	128.1035	128.1027	128.1054
$\hat{\bar{Y}}_2$	120.5440	120.5467	120.5452	120.5436	120.5428	120.5454
$\hat{\bar{Y}}_3$	111.9052	111.9077	111.9064	111.9048	111.9041	111.9065
$\hat{\bar{Y}}_4$	115.8998	115.9024	115.9010	115.8994	115.8986	115.9011
$\hat{\bar{Y}}_5$	110.6168	110.6194	110.6180	110.6165	110.6157	110.6181
$\hat{\bar{Y}}_6$	105.4510	105.4534	105.4521	105.4507	105.4500	105.4522
$\hat{\bar{Y}}_7$	124.3288	124.3317	124.3301	124.3284	124.3276	124.3303
$\hat{\bar{Y}}_8$	117.3310	117.3337	117.3322	117.3306	117.3298	117.3323
$\hat{\bar{Y}}_9$	109.6899	109.6924	109.6910	109.6895	109.6888	109.6911
$\hat{\bar{Y}}_{10}$	119.6631	119.6658	119.6644	119.6627	119.6619	119.6645
$\hat{\bar{Y}}_{11}$	113.5341	113.5366	113.5352	113.5337	113.5329	113.5353
$\hat{\bar{Y}}_{12}$	107.2274	107.2298	107.2285	107.2270	107.2263	107.2286

CONCLUSION

From the above results it can be concluded that the proposed ratio estimators are more efficient than the existing estimators and the above estimators are also more efficient than the classical ratio estimator, thus providing better alternative estimators for use in practical situations.

REFERENCES

Cochran, W. G. (1940). The estimation of the yields of the cereal experiments by sampling for the ratio of grain to total produce. *The Journal of Agricultural Science, 30*, 262-275.

Cochran, W. G. (1977). *Sampling Techniques*, John Wiley and Sons, New York.

Kadilar, C., & Cingi, H. (2006). An Improvement in Estimating the Population mean by using the Correlation Coefficient. *Hacettepe Journal of Mathematics and Statistics, 35*(1), 103-109.

Kadilar, C., & Cingi, H. (2004). Ratio estimators in simple random sampling.*Applied Mathematics and Computation,151*, 893-902.

Koyuncu, N., & Kadilar, C. (2009). Efficient Estimators for the Population mean. *Hacettepe Journal of Mathematics and Statistics*, *38*(2), 217-225.

Murthy, M. N. (1967). *Sampling theory and methods*, Statistical Publishing Society, Calcutta, India.

Prasad, B. (1989). Some improved ratio type estimators of population mean and ratio in finite population sample surveys. *Communications in Statistics: Theory and Methods, 18*, 379–392.

Rao, T.J. (1991). On certain methods of improving ratio and regression estimators.*Communications in Statistics: Theory and Methods, 20*(10), 3325–3340.

Singh, D., & Chaudhary, F.S. (1986). *Theory and Analysis of Sample Survey Designs*, New Age International Publisher.

Singh, G.N. (2003). On the improvement of product method of estimation in sample surveys. *Journal of the Indian Society of Agricultural Statistics, 56*(3), 267–265.

Singh, H.P., & Tailor, R. (2003). Use of known correlation coefficient in estimating the finite population means. *Statistics in Transition – new series, 6*(4), 555-560.

Singh, H.P., & Tailor, R. (2005). Estimation of finite population mean with known coefficient of variation of an auxiliary. *STATISTICA, anno LXV, 3*, 301-310.

Singh, H.P., Tailor, R., Tailor, R., & Kakran, M.S. (2004). An Improved Estimator of population means using Power transformation. *Journal of the Indian Society of Agricultural Statistics, 58*(2), 223-230.

Sisodia, B.V.S., & Dwivedi, V.K. (1981). A modified ratio estimator using coefficient of variation of auxiliary variable. *Journal of the Indian Society of Agricultural Statistics, 33*(1), 13-18.

Subramani, J., & Kumarapandiyan, G. (2012). A class of modified ratio estimators using deciles of an auxiliary variable. *International Journal of Statistical Application, 2*, 101-107.

Subzar, M., Abid, M., Maqbool, S., Raja, T. A., Shabeer, M., & Lone, B. A. (2017). A Class of Improved Ratio Estimators for Population Mean using Conventional Location parameters. *International Journal of Modern Mathematical Sciences,15*(2), 187-205.

Subzar, M., Maqbool, S., Raja, T. A., & Shabeer, M. (2017). A New Ratio Estimators for estimation of Population mean using Conventional Location parameters. *World Applied Sciences Journal, 35*(3), 377-384.

Subzar, M., Raja, T. A., Maqbool, S., & Nazir, N. (2016). New Alternative to Ratio Estimator of Population Mean. *International Journal of Agricultural Statistical Sciences,12*(1), 221-225.

Subzar, M., Raja, T. A., Maqbool, S., Rishu, J., & Shabeer, M. (2017). Enhancing the Ratio Type Estimators for Estimating Population Mean using Auxiliary Information in Survey Sampling. *International Journal of Agricultural Statistical Sciences, 13*(1), 213-220.

Upadhyaya, L.N., & Singh, H.P. (1999). Use of transformed auxiliary variable in estimating the finite population means. *Biometrical Journal, 41*(5), 627-636.

Wolter K.M. (1985). *Introduction to Variance Estimation,*Springer-Verlag.

Yan, Z., & Tian, B. (2010). Ratio Method to the Mean Estimation Using Coefficient of Skewness of Auxiliary Variable.*ICICA, Part II, CCIS,106,* 103–110.